Shape Modeling: Curves and Surfaces

Course web page:

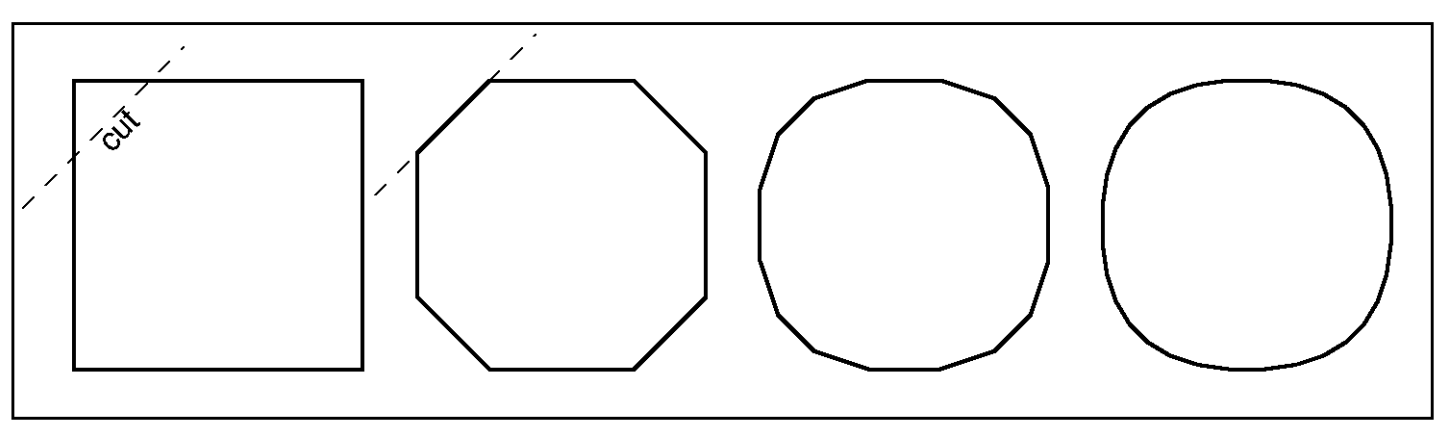
http://goo.gl/EB3aA

Outline

- Subdivision
- Geometry shaders
- Pretty pictures

Curve Subdivision

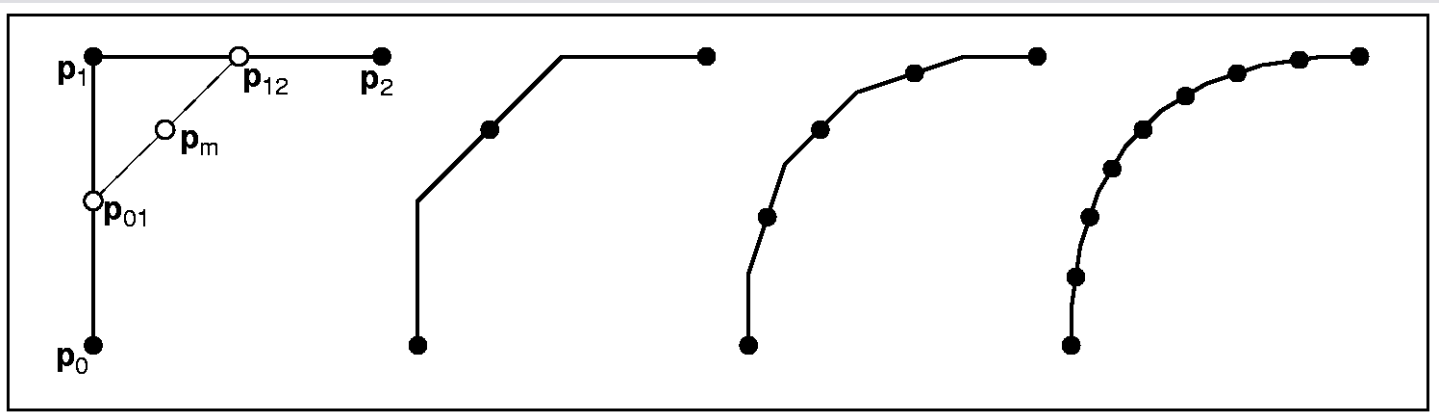
- Goal: Algorithmically obtain smooth curves starting from small number of line segments
- One approach: **Corner-cutting** subdivision
  - Repeatedly chop off corners of polygon
  - Each line segment is replaced by two shorter segments
  - **Limit curve** is shape that would be reached after an infinite series of such subdivisions



from Shirley

Midpoint Corner-cutting Algorithm

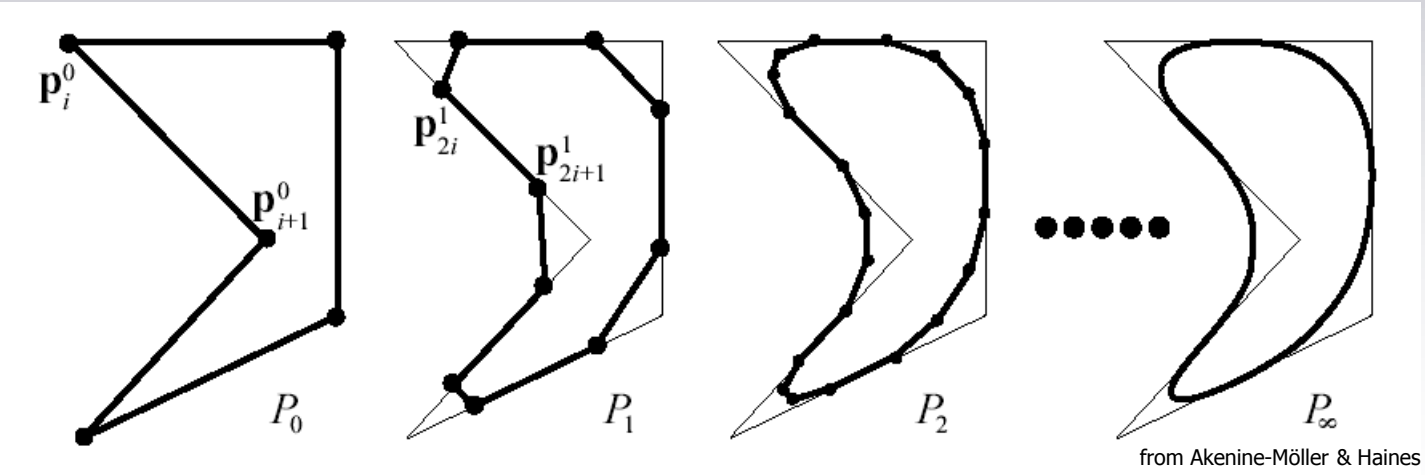
```
function midpoint_subdivide(p0, p1, p2, depth)
{
   if (depth > 0) {
       p01 = (p0 + p1)/2;
       p12 = (p1 + p2)/2;
       pm = (p01 + p12)/2;
       midpoint_subdivide(p0, p01, pm, depth - 1);
       midpoint_subdivide(pm, p12, p2, depth - 1);
   } // else draw
}
```



from Shirley

Bézier curves and B-splines via subdivision

- The midpoint corner-cutting algorithm is a subdivision definition of quadratic Bézier curves
- **Chaikin**'s subdivision scheme defines quadratic B-splines
  - Make new edge joining point ¾ of way to next control point $\mathbf{p}_i$ with point ¼ of way after $\mathbf{p}_i$

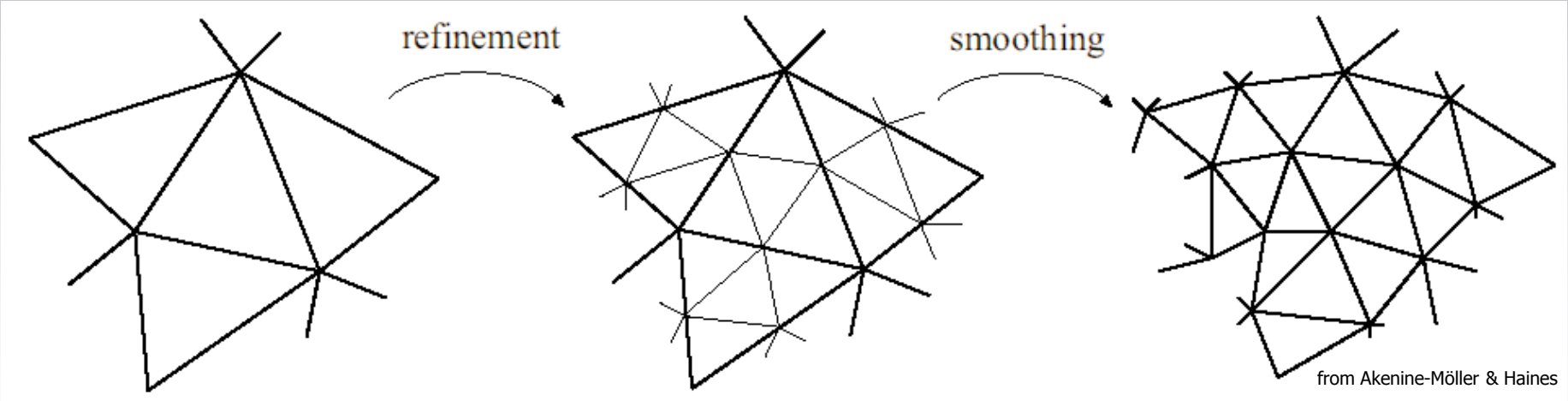


from Akenine-Möller & Haines

Chaikin's scheme

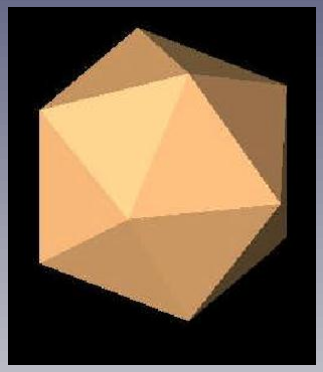
Surface Subdivision

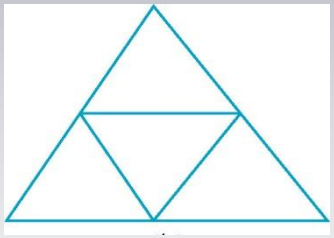
- Analogous to curve subdivision:
  1. **Refine mesh**: Choose new vertices to make smaller polygons, update connectivity
  2. **Smooth mesh**: Move vertices to fit underlying object



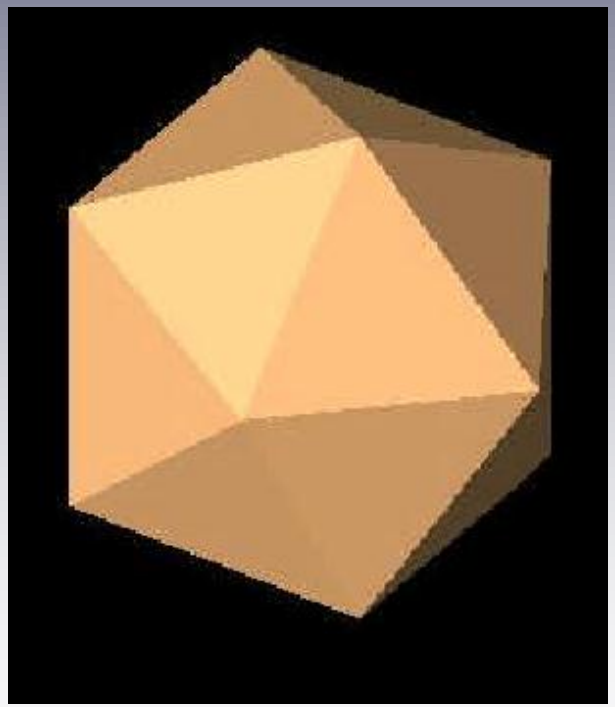
Subdivision for Sphere Definition

- Start with icosahedron with triangular faces
  - Compute midpoint of each edge of triangle
    - This defines 4 new triangles
  - Renormalize midpoints (new vertices) so that they lie on sphere
    - This makes original triangle non-planar
  - Recurse on each new face until a desired depth is reached
    - At leaf of recursion, draw triangle

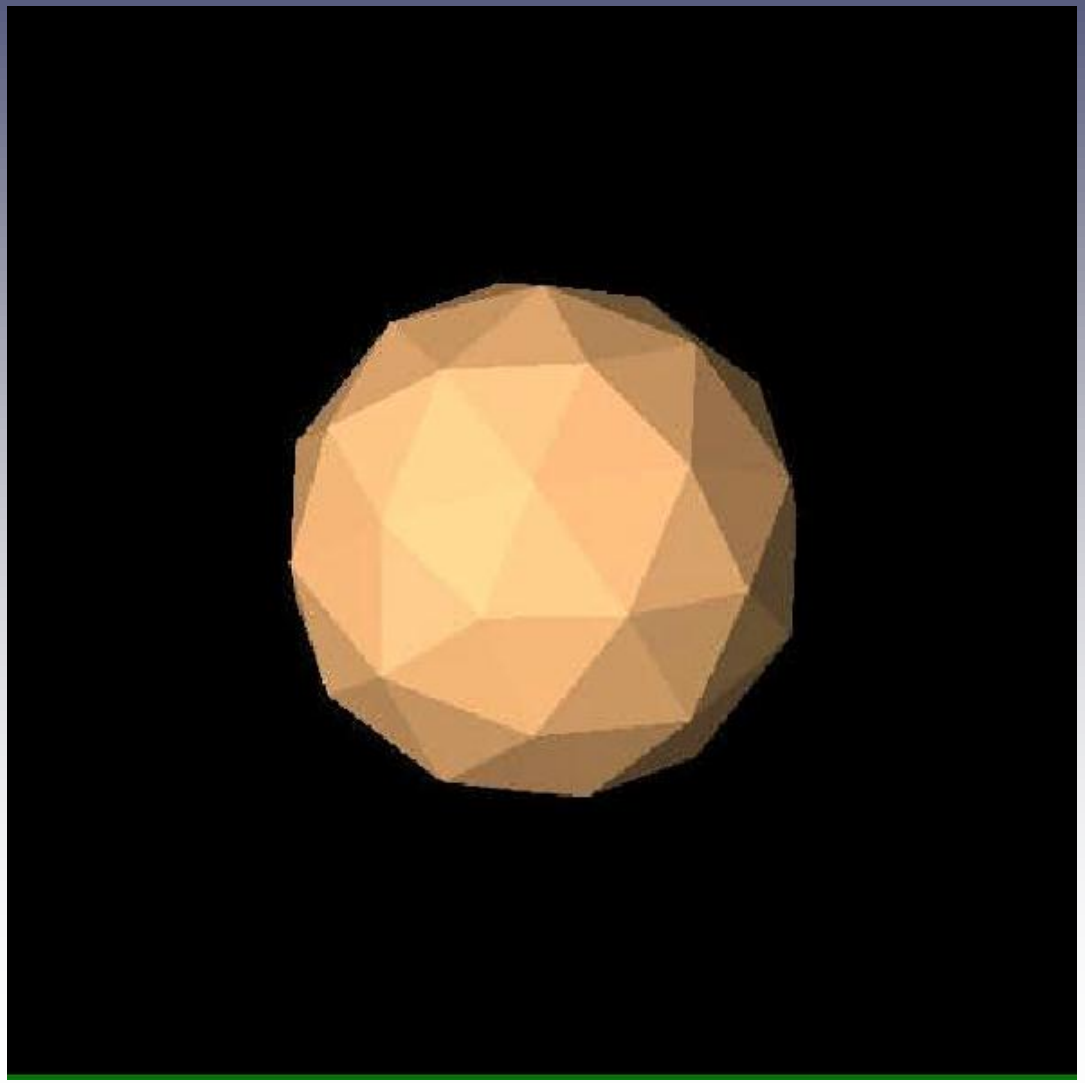




Step 0

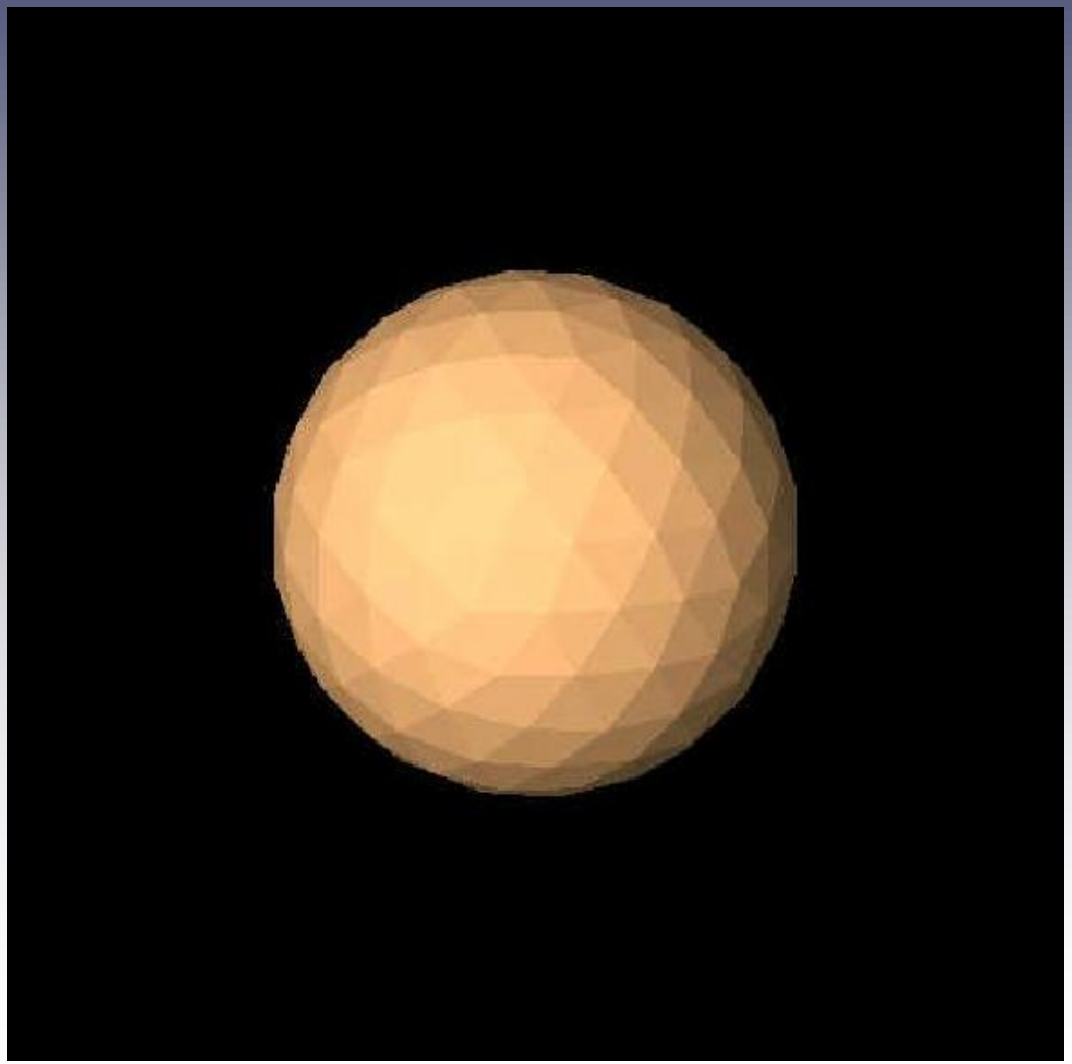


Step 1

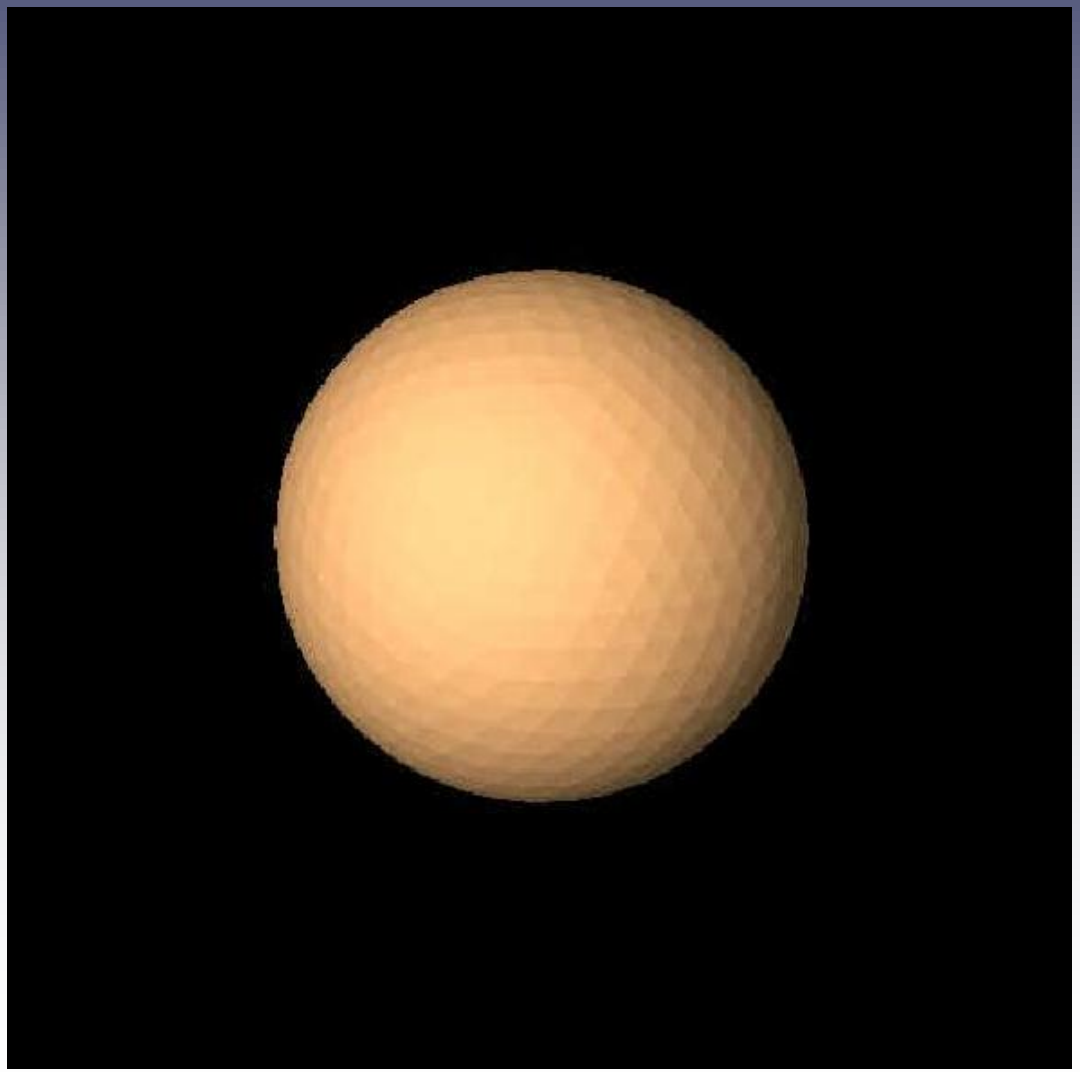


courtesy of F. Pfenning

Step 2

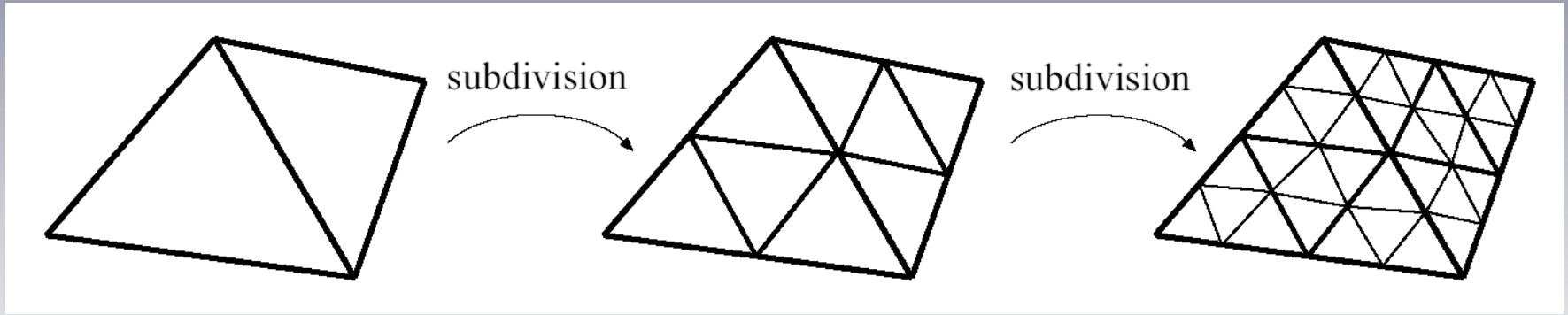


Step 3



Loop subdivision

- Smooths **triangle** mesh
- Subdivision replaces 1 triangle with 4

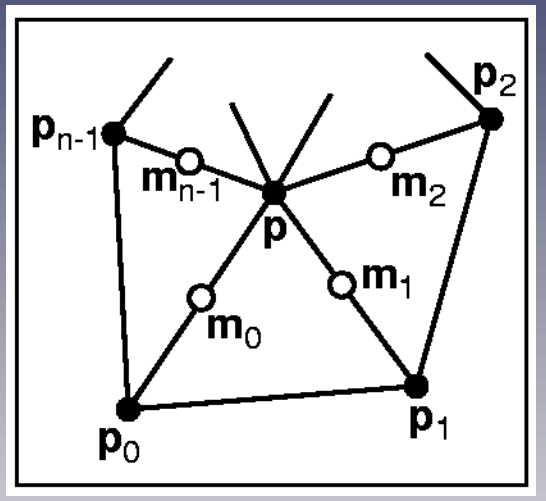


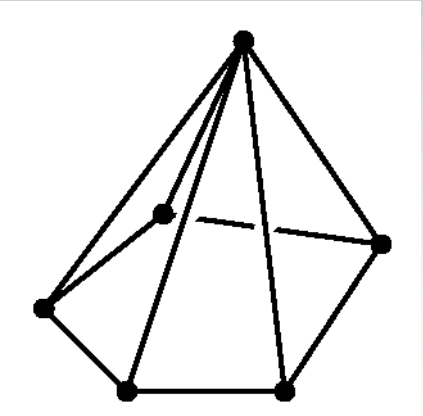
from Akenine-Möller & Haines

- Approximating scheme
  - Original vertices not guaranteed to be in subdivided mesh

Loop subdivision: Algorithm

- Consider a point **p** in the mesh connected to n other vertices $\mathbf{p}_0, \ldots, \mathbf{p}_{n-1}$
- Regard **p** as apex of pyramid defined by itself and connected vertices
- Idea is to smooth pointy pyramid by rounding it off
  - Lower apex, "bend" diagonals at midpoints

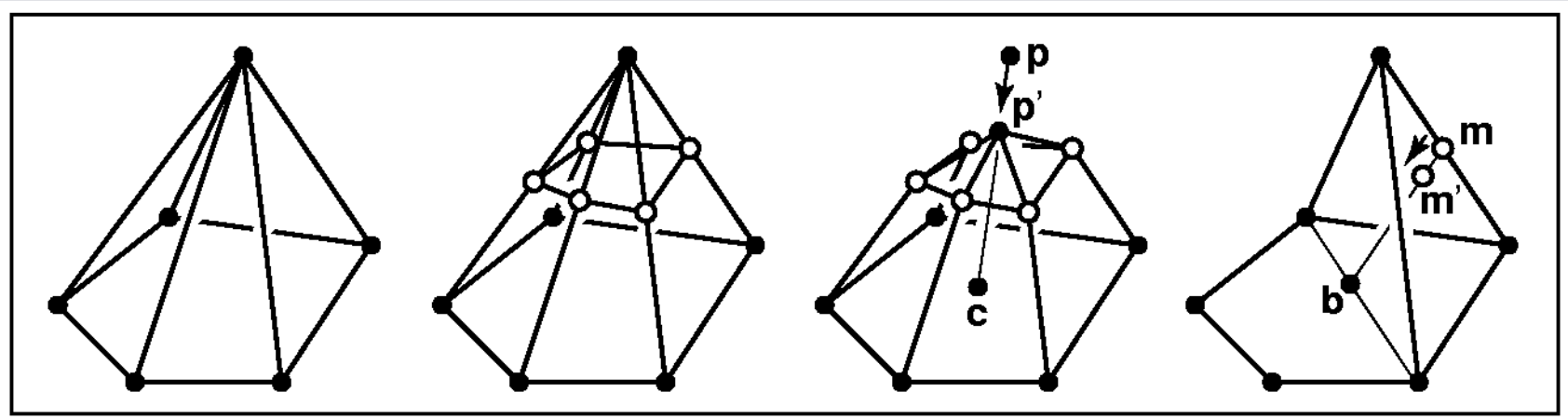




from Shirley

Loop subdivision: Algorithm

- Move each vertex **p** some fraction **s** of the distance toward centroid **c** of polygon defined by surrounding vertices
- Create and move each midpoint $\mathbf{m}_i$ between **p** and connected vertex $\mathbf{p}_i$ some fraction **t** toward midpoint **b** of $\mathbf{p}_{i-1}$ of and $\mathbf{p}_{i+1}$



Loop subdivision: Notes

- It was proved that using t = 0.25 and

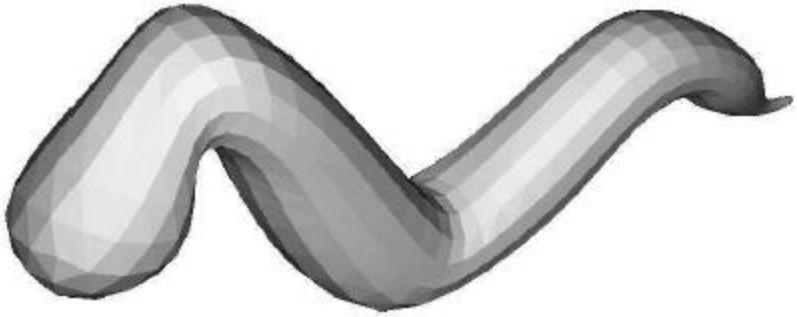
$$s = \frac{5}{8} - \frac{[3 + 2\cos(\frac{2\pi}{n})]^2}{64}$$

gives C^2 continuity at regular (6-connected) vertices and C^1 elsewhere

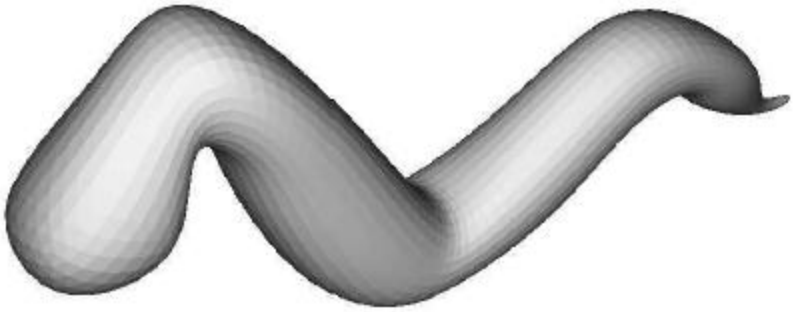
Loop subdivision: Example

Original points

After 1 subdivision



After 2 subdivisions



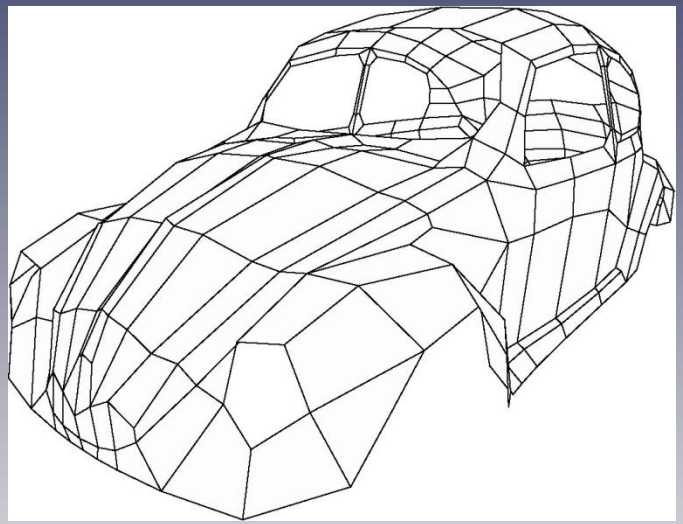
After 3 subdivisions

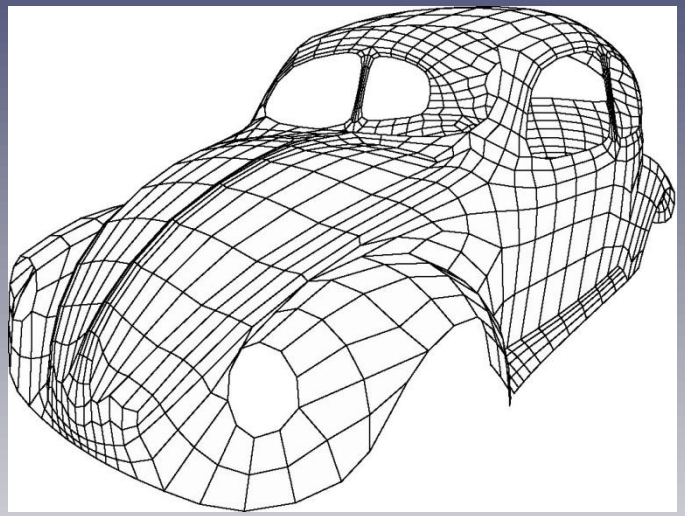
from Akenine-Möller & Haines

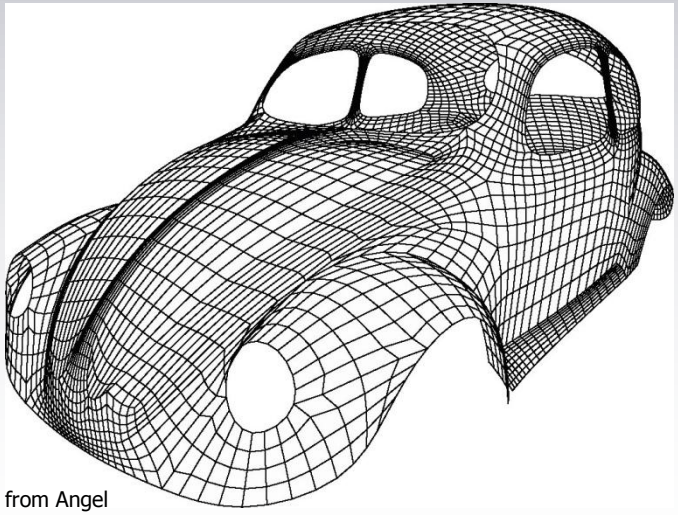
Other surface subdivision schemes

- Triangle
  - sqrt(3): Splits each triangle into 3 instead of 4 subtriangles
  - Modified butterfly: Interpolating subdivision (vs. approximating of Loop)
- Polygonal (i.e., quadrilaterals—easier for texture mapping)
  - Catmull-Clark

Subdivision: Example (Catmull-Clark)

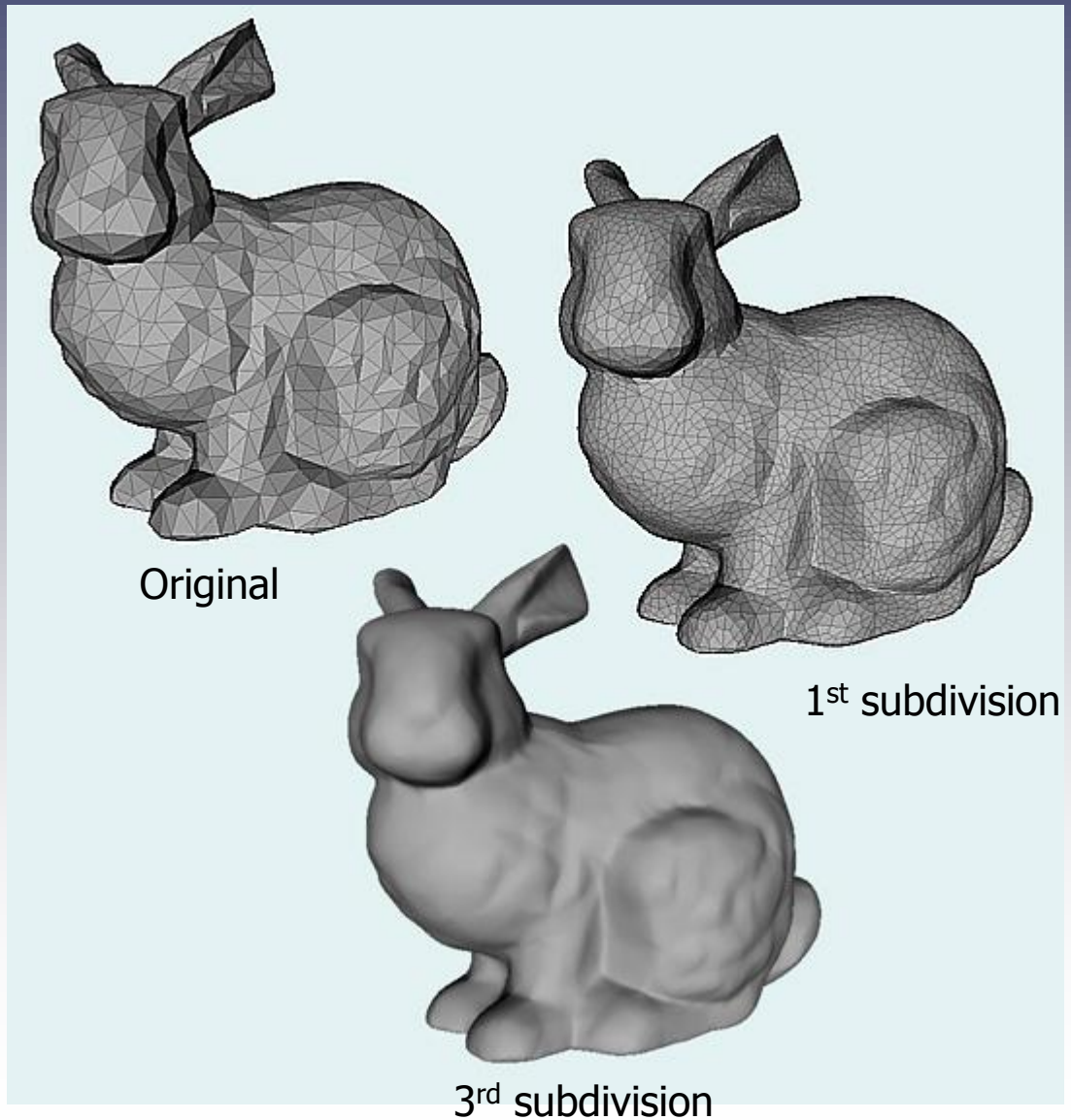






from Angel

Subdivision: Example (Catmull-Clark)



courtesy of E. Pasztor

Geometry Shaders (since OpenGL 3.2)

- Take primitive as input (possibly with adjacency information)
  - Point, line, triangle
- Generate zero or more new primitives
- Run after vertex shaders
- Perfect for GPU-based tessellation, subdivision
- See
  http://www.lighthouse3d.com/tutorials/glsl-core-tutorial/geometry-shader/
  - http://www.youtube.com/watch?v=_ZTwSVKCJgg

Videos

- SIGGRAPH 2009, 2010, 2011 video previews
  - http://www.youtube.com/watch?v=qC5Y9W-E-po (2:57)
  - http://www.youtube.com/watch?v=-PMf3XrwPKo (3:45)
  - http://www.youtube.com/watch?v=JK9EEE3RsKM (3:28)