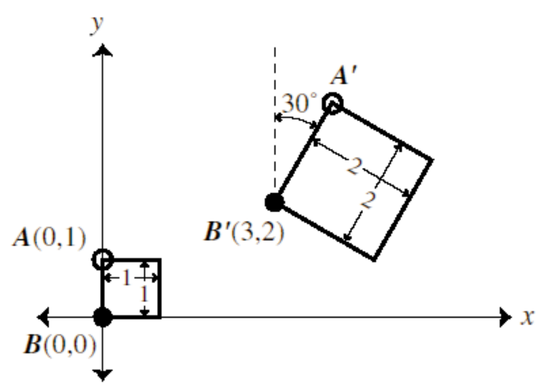
CISC 440/640

Midterm

Thursday, October 28, 2004

*Only graduate students are required to answer questions **2.2** and **4.4** of this exam. Point totals will be normalized to a common scale (the exam is worth 15% of your grade). Partial credit will be given in half-point gradations.*

Part 1



1.1 (2 points)

The above diagram shows a 2-D transformation applied to a unit square. The overall transformation **T** can be described in terms of a number of simpler transformations. Name each of these simple transformations, give a matrix representation of each using homogeneous coordinates, and specify the order in which they are applied.

1.2 (1 point)

Apply the transformation $\mathbf{T}$ you have specified to find the (x, y) coordinates of point A', the image of point A under the overall transformation. Show your work (you do not have to evaluate trigonometric expressions).

Part 2

2.1 (1 point)

Suppose you are rasterizing a triangle whose corners in image coordinates are at $(1, 1)$, $(5, 1)$, and $(2, 9)$, and the z depths of those corners are 22, 29, and 25, respectively. Using the bilinear interpolation scheme of z-buffering, what is the depth of the interior point at $(3, 5)$?

2.2 Graduate students only (3 points)

Explain in as much detail as possible why the midpoint algorithm is a more efficient line rasterizer than DDA.

Part 3

3.1 (2 points)

What is the major difference between global illumination models and local illumination models such as the Phong model we have been studying? List at least three visual phenomena that local models do not capture.

3.2 (3 points)

The *reflectance equation* is

$$L_o(\mathbf{x}, \theta_o, \phi_o) = \int_\Omega f(\theta_o, \phi_o, \theta_i, \phi_i) L_i(\mathbf{x}, \theta_i, \phi_i) \cos\theta_i d\omega$$

Explain what this formula represents and identify each of its terms (both the functions and the variables).

Part 4

4.1 (2 points)

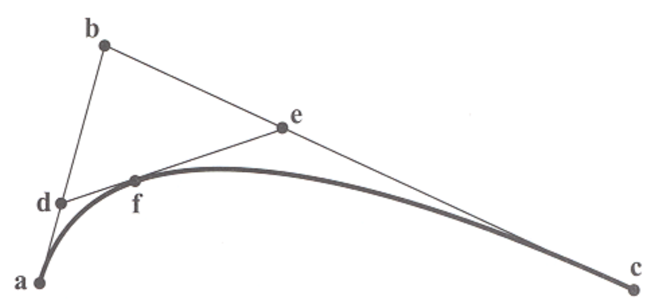
Define *blending function* and *control point* in the context of parametric line segments and curves or splines. Why are parametric curves and surfaces a useful object representation?

4.2 (1 point)

What's the difference between an *interpolating* spline and an *approximating* one? Give an example of each for either curves or surfaces.

4.3 (3 points)

How many control points and blending functions are necessary to describe a quadratic Bézier curve? Using the idea of recursive linear interpolation as shown geometrically in the figure below, give an algebraic derivation of the quadratic Bézier curve blending functions. Use the variable names in the figure–i.e., express the point on the curve **f** in terms of the original control points and t. Sketch and label the blending functions over the interval $t \in [0, 1]$.



4.4 Graduate students only (2 points)

A Catmull-Rom spline curve is a cubic polynomial defined parametrically over the interval $t \in [0, 1]$. It is of the form $\mathbf{P}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$. Solve for the unknown values of a_0, a_1, a_2, a_3 in terms of the known positions at the curve's start $\mathbf{P}(0)$ and end $\mathbf{P}(1)$ as well as the known starting tangent $\mathbf{P}'(0)$ and ending tangent $\mathbf{P}'(1)$.