13.4 Odometry

Odometry is the process of estimating the chassis configuration q from the wheel motions, essentially integrating the effect of the wheel velocities. Since wheel-rotation sensing is available on all mobile robots, odometry is cheap and convenient. Estimation errors tend to accumulate over time, though, due to unexpected slipping and skidding of the wheels and to numerical integration error. Therefore, it is common to supplement odometry with other position sensors, such as GPS, the visual recognition of landmarks, ultrasonic beacons, laser or ultrasonic range sensing, etc. Those sensing modalities have their own measurement uncertainty but errors do not accumulate over time. As a result, odometry generally gives superior results on short time scales, but odometric estimates should either (1) be periodically corrected by other sensing modalities or, preferably, (2) integrated with other sensing modalities in an estimation framework based on a Kalman filter, particle filter, or similar.

In this section we focus on odometry. We assume that each wheel of an omnidirectional robot, and each rear wheel of a diff-drive or car, has an encoder that senses how far the wheel has rotated in its driving direction. If the wheels are driven by stepper motors then we know the driving rotation of each wheel from the steps we have commanded to it.

The goal is to estimate the new chassis configuration q_{k+1} as a function of the previous chassis configuration q_k, given the change in wheel angles from the instant k to the instant $k+1$.

Let $\Delta\theta_i$ be the change in wheel i's driving angle since the wheel angle was last queried a time Δt ago. Since we know only the net change in the wheel driving angle, not the time history of how the wheel angle evolved during the time interval, the simplest assumption is that the wheel's angular velocity was constant during the time interval, $\dot{\theta}_i = \Delta\theta_i/\Delta t$. The choice of units used to measure the time interval is not relevant (since we will eventually integrate the chassis body twist $\mathcal{V}_b$ over the same time interval), so we set $\Delta t = 1$, i.e., $\dot{\theta}_i = \Delta\theta$.

For omnidirectional mobile robots, the vector of wheel speeds $\dot{\theta}$, and therefore $\Delta\theta$, is related to the body twist $\mathcal{V}_b = (\omega_{bz}, v_{bx}, v_{by})$ of the chassis by Equation (13.8):

$$\Delta\theta = H(0)\mathcal{V}_b,$$

where $H(0)$ for the three-omniwheel robot is given by Equation (13.9) and for the four-mecanum-wheel robot is given by Equation (13.10). Therefore, the body twist $\mathcal{V}_b$ corresponding to $\Delta\theta$ is

$$\mathcal{V}_b = H^{\dagger}(0)\Delta\theta = F\Delta\theta,$$

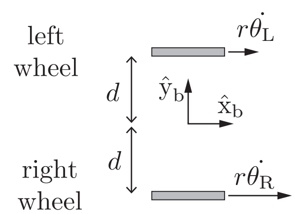

Figure 13.22: The left and right wheels of a diff-drive or the left and right rear wheels of a car.

where $F = H^\dagger(0)$ is the pseudoinverse of $H(0)$. For the three-omniwheel robot,

$$\mathcal{V}_b = F\Delta\theta = r \begin{bmatrix} -1/(3d) & -1/(3d) & -1/(3d) \\ 2/3 & -1/3 & -1/3 \\ 0 & -1/(2\sin(\pi/3)) & 1/(2\sin(\pi/3)) \end{bmatrix} \Delta\theta \tag{13.32}$$

and for the four-mecanum-wheel robot,

$$\mathcal{V}_b = F\Delta\theta = \frac{r}{4} \begin{bmatrix} -1/(\ell+w) & 1/(\ell+w) & 1/(\ell+w) & -1/(\ell+w) \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \Delta\theta. \tag{13.33}$$

The relationship $\mathcal{V}_b = F\dot{\theta} = F\Delta\theta$ also holds for the diff-drive robot and the car (Figure 13.22), where $\Delta\theta = (\Delta\theta_\mathrm{L}, \Delta\theta_\mathrm{R})$ (the increments for the left and right wheels) and

$$\mathcal{V}_b = F\Delta\theta = r \begin{bmatrix} -1/(2d) & 1/(2d) \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\theta_\mathrm{L} \\ \Delta\theta_\mathrm{R} \end{bmatrix}. \tag{13.34}$$

Since the wheel speeds are assumed constant during the time interval, so is the body twist $\mathcal{V}_b$. Calling $\mathcal{V}_{b6}$ the six-dimensional version of the planar twist $\mathcal{V}_b$ (i.e., $\mathcal{V}_{b6} = (0, 0, \omega_{bz}, v_{bx}, v_{by}, 0)$), $\mathcal{V}_{b6}$ can be integrated to generate the displacement created by the wheel-angle increment vector $\Delta\theta$:

$$T_{bb'} = e^{[\mathcal{V}_{b6}]}.$$

From $T_{bb'} \in SE(3)$, which expresses the new chassis frame $\{\mathrm{b}'\}$ relative to the initial frame $\{\mathrm{b}\}$, we can extract the change in coordinates relative to the body frame {b}, $\Delta q_b = (\Delta\phi_b, \Delta x_b, \Delta y_b)$, in terms of $(\omega_{bz}, v_{bx}, v_{by})$:

$$\text{if } \omega_{bz} = 0, \qquad \Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} 0 \\ v_{bx} \\ v_{by} \end{bmatrix}; \tag{13.35}$$

$$\text{if } \omega_{bz} \neq 0, \qquad \Delta q_b = \begin{bmatrix} \Delta\phi_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix} = \begin{bmatrix} \omega_{bz} \\ (v_{bx}\sin\omega_{bz} + v_{by}(\cos\omega_{bz} - 1))/\omega_{bz} \\ (v_{by}\sin\omega_{bz} + v_{bx}(1 - \cos\omega_{bz}))/\omega_{bz} \end{bmatrix}.$$

Transforming Δq_b in {b} to Δq in the fixed frame {s} using the chassis angle ϕ_k,

$$\Delta q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_k & -\sin\phi_k \\ 0 & \sin\phi_k & \cos\phi_k \end{bmatrix} \Delta q_b, \tag{13.36}$$

the updated odometry estimate of the chassis configuration is finally

$$q_{k+1} = q_k + \Delta q.$$

In summary, Δq is calculated using Equations (13.35) and (13.36) as a function of $\mathcal{V}_b$ and the previous chassis angle ϕ_k, and Equation (13.32), (13.33), or (13.34) is used to calculate $\mathcal{V}_b$ as a function of the wheel-angle changes $\Delta\theta$ for the three-omniwheel robot, the four-mecanum-wheel robot, or a nonholonomic robot (the diff-drive or the car), respectively.

13.5 Mobile Manipulation

For a robot arm mounted on a mobile base, **mobile manipulation** describes the coordination of the motion of the base and the robot joints to achieve a desired motion at the end-effector. Typically the motion of the arm can be controlled more precisely than the motion of the base, so the most popular type of mobile manipulation involves driving the base, parking it, letting the arm perform the precise motion task, then driving away.

In some cases, however, it is advantageous, or even necessary, for the end-effector motion to be achieved by a combination of motion of the base and motion of the arm. Defining the fixed space frame {s}, the chassis frame {b}, a frame at the base of the arm {0}, and an end-effector frame {e}, the configuration of {e} in {s} is

$$X(q,\theta) = T_{se}(q,\theta) = T_{sb}(q)\, T_{b0}\, T_{0e}(\theta) \in SE(3),$$