Scan Matching

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Scan Matching Overview

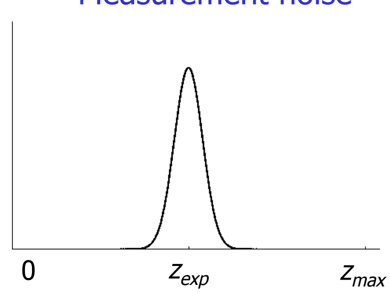
- Problem statement:
  - Given a scan and a map, or a scan and a scan, or a map and a map, find the rigid-body transformation (translation+rotation) that aligns them best
- Benefits:
  - Improved proposal distribution (e.g., gMapping)
  - Scan-matching objectives, even when not meaningful probabilities, can be used in graphSLAM / pose-graph SLAM (see later)
- Approaches:
  - Optimize over x: p(z | x, m), with:
    - 1. p(z | x, m) = beam sensor model --- sensor beam full readings <-> map
    - 2. p(z | x, m) = likelihood field model --- sensor beam endpoints <-> likelihood field
    - 3. p(m_{local} | x, m) = map matching model --- local map <-> global map
  - Reduce both entities to a set of points, align the point clouds through the Iterative Closest Points (ICP)
    - 4. cloud of points <-> cloud of points --- sensor beam endpoints <-> sensor beam endpoints
- Other popular use (outside of SLAM): pose estimation and verification of presence for objects detected in point cloud data

Outline

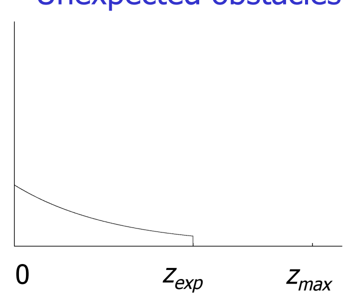
- **1. Beam Sensor Model**
- 2. Likelihood Field Model
- 3. Map Matching
- 4. Iterated Closest Points (ICP)

Beam-based Proximity Model

Measurement noise



Unexpected obstacles



$$P_{hit}(z \mid x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2}\frac{(z - z_{\exp})^2}{b}}$$

$$P_{\text{unexp}}(z \mid x, m) = \begin{cases} \eta \, \lambda \, e^{-\lambda z} & z < z_{\exp} \\ 0 & \textit{otherwise} \end{cases}$$

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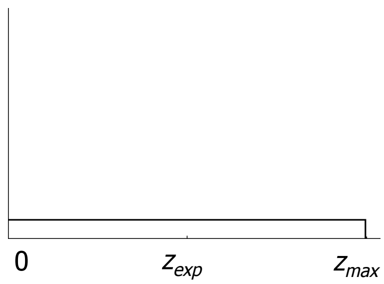
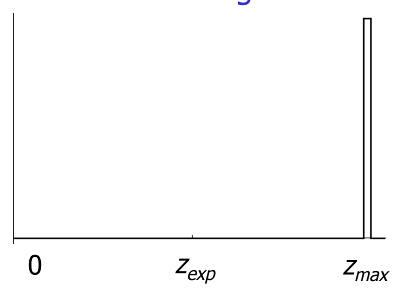
Beam-based Proximity Model

Random measurement



$$P_{rand}(z \mid x, m) = \eta \frac{1}{z_{\max}}$$

Max range



$$P_{\max}(z \mid x, m) = \eta \frac{1}{z_{small}}$$

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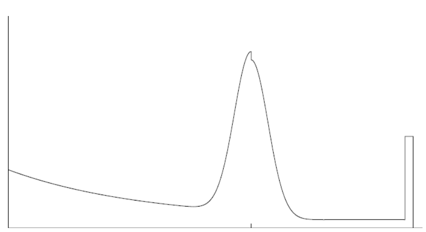
Resulting Mixture Density



$$P(z \mid x, m) = \begin{pmatrix} \alpha_{\mathrm{hit}} \\ \alpha_{\mathrm{unexp}} \\ \alpha_{\max} \\ \alpha_{\mathrm{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\mathrm{hit}}(z \mid x, m) \\ P_{\mathrm{unexp}}(z \mid x, m) \\ P_{\max}(z \mid x, m) \\ P_{\mathrm{rand}}(z \mid x, m) \end{pmatrix}$$

How can we determine the model parameters?

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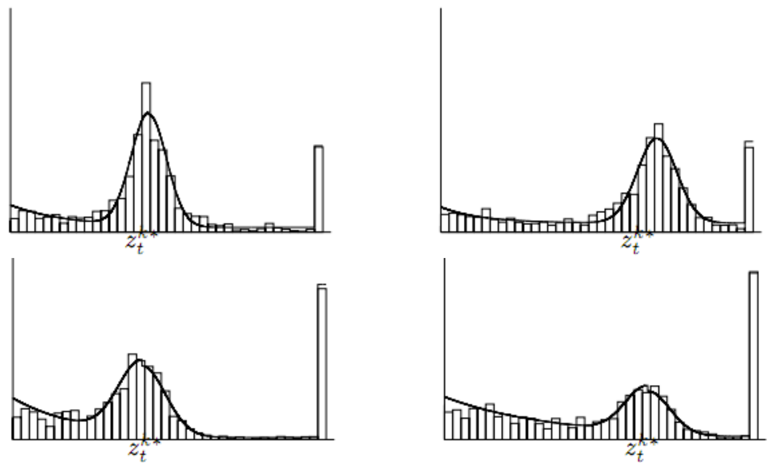
Approximation Results

Laser



300cm

400cm

Sonar

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Summary Beam Sensor Model

- Assumes independence between beams.
  - Justification?
  - Overconfident!
- Models physical causes for measurements.
  - Mixture of densities for these causes.
  - Assumes independence between causes. Problem?
- Implementation
  - Learn parameters based on real data.
  - Different models should be learned for different angles at which the sensor beam hits the obstacle.
  - Determine expected distances by ray-tracing.
  - Expected distances can be pre-processed.

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Drawbacks Beam Sensor Model

- Lack of smoothness
  - $P(z \mid x_t, m)$ is not smooth in x_t
  - Problematic consequences:
    - For sampling based methods: nearby points have very different likelihoods, which could result in requiring large numbers of samples to hit some "reasonably likely" states
    - Hill-climbing methods that try to find the locally most likely x_t have limited abilities per many local optima
- Computationally expensive
  - Need to ray-cast for every sensor reading
  - Could pre-compute over discrete set of states (and then interpolate), but table is large per covering a 3-D space and in SLAM the map (and hence table) change over time

Outline

- 1. Beam Sensor Model
- **2. Likelihood Field Model**
- 3. Map Matching
- 4. Iterated Closest Points (ICP)

Likelihood Field Model

aka Beam Endpoint Model aka Scan-based Model

- Overcomes lack-of-smoothness and computational limitations of Sensor Beam Model
- Ad-hoc algorithm: not considering a conditional probability relative to any meaningful generative model of the physics of sensors
- Works well in practice.
- Idea: Instead of following along the beam (which is expensive!) just check the end-point. The likelihood $p(z \mid x_t, m)$ is given by:

$$\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{d^2}{2\sigma^2})$$

with d = distance from end-point to nearest obstacle.

Algorithm: likelihood_field_range_finder_model(z_t, x_t, m)

1. $q = 1$
2. for all k do
3. $\quad$ if $z_t^k \neq z_{\max}$
4. $\quad\quad x_{z_t^k} = x + x_{k,\text{sens}} \cos\theta - y_{k,\text{sens}} \sin\theta + z_t^k \cos(\theta + \theta_{k,\text{sens}})$
5. $\quad\quad y_{z_t^k} = y + y_{k,\text{sens}} \cos\theta - x_{k,\text{sens}} \sin\theta + z_t^k \sin(\theta + \theta_{k,\text{sens}})$
6. $\quad\quad d = \min_{x',y'}\{(x_{z_t^k} - x')^2 + (y_{z_t^k} - y')^2 | (x', y') \text{is occupied in } m\}$
7. $\quad\quad q = q \cdot (p_{\text{hit}} \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{d^2}{2\sigma^2}) + p_{\text{random}} \frac{1}{z_{\max}})$
8. return q

In practice: pre-compute "likelihood field" over (2-D) grid.

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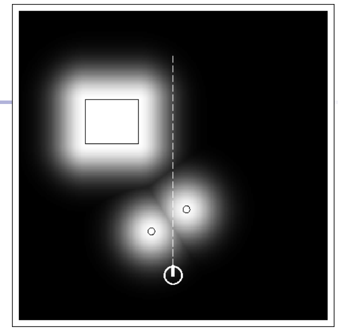
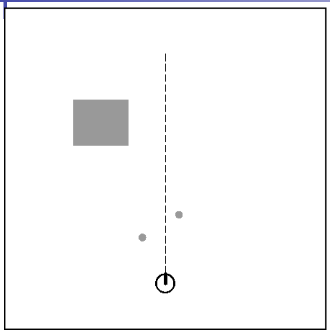
Example

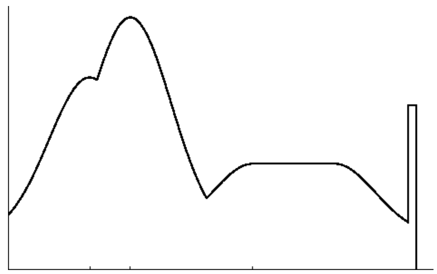


Likelihood field



Map m

$P(z|x,m)$



Note: "p(z|x,m)" is not really a density, as it does not normalize to one when integrating over all z

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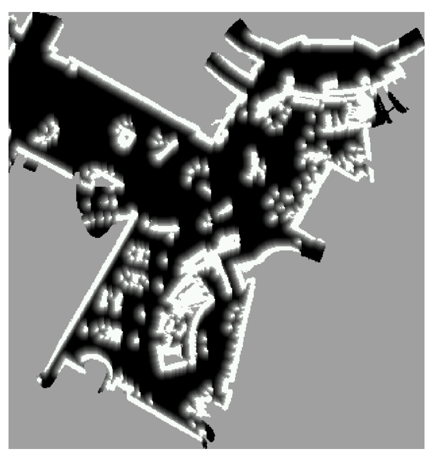
San Jose Tech Museum



Occupancy grid map

Likelihood field

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Drawbacks of Likelihood Field Model

- No explicit modeling of people and other dynamics that might cause short readings
- No modeling of the beam --- treats sensor as if it can see through walls
- Cannot handle unexplored areas
  - Fix: when endpoint in unexplored area,

    have $p(z_t \mid x_t, m) = 1 / z_{max}$

Scan Matching

- As usual, maximize over x_t the likelihood $p(z_t \mid x_t, m)$
- The objective $p(z_t \mid x_t, m)$ now corresponds to the likelihood field based score

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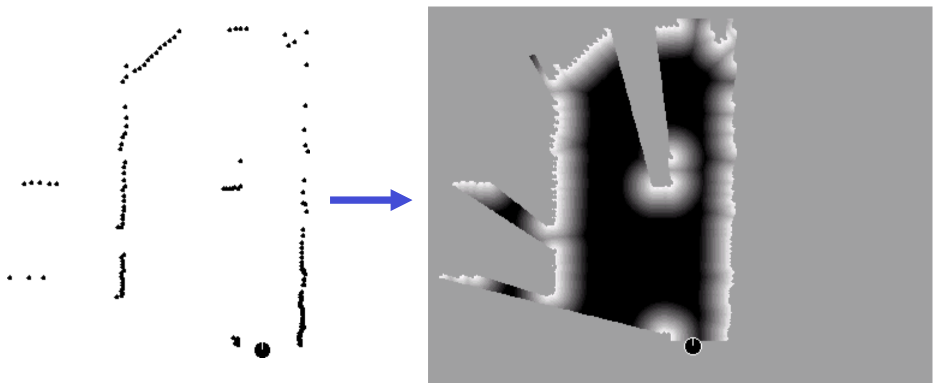
Scan Matching

- Can also match two scans: for first scan extract likelihood field (treating each beam endpoint as occupied space) and use it to match the next scan. [can also symmetrize this]



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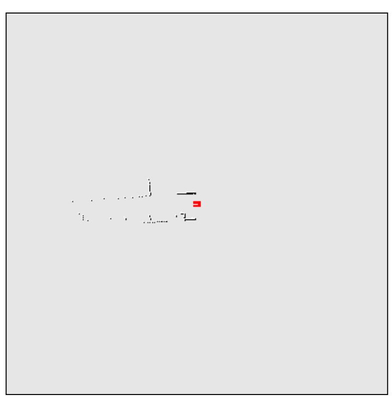
Scan Matching

- Extract likelihood field from first scan and use it to match second scan.



~0.01 sec

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Properties of Scan-based Model

- Highly efficient, uses 2D tables only.
- Smooth w.r.t. to small changes in robot position.
- Allows gradient descent, scan matching.
- Ignores physical properties of beams.

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Map Matching

- Generate small, local maps from sensor data and match local maps against global model.
- Correlation score:

$$\rho_{m,m_{\text{local}},x_t} = \frac{\sum_{x,y}(m_{x,y}-\bar{m})\cdot(m_{x,y,\text{local}}(x_t)-\bar{m})}{\sqrt{\sum_{x,y}(m_{x,y}-\bar{m})^2}\sqrt{\sum_{x,y}(m_{x,y,\text{local}}(x_t)-\bar{m})^2}}$$

with $\bar{m} = \frac{1}{2N}\sum_{x,y}(m_{x,y}+m_{x,y,\text{local}})$

- Likelihood interpretation:

$$p(m_{\text{local}}|x_t,m) = \max\{\rho_{m,m_{\text{local}},x_t},0\}$$

- To obtain smoothness: convolve the map m with a Gaussian, and run map matching on the smoothed map

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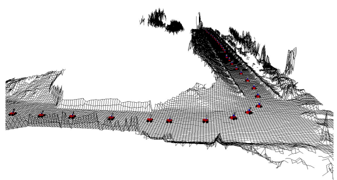
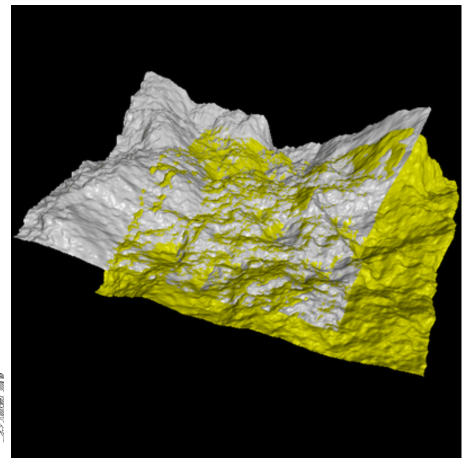
Motivation





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Known Correspondences

- Given: two corresponding point sets:

$$X = \{x_1, ..., x_n\}$$

$$P = \{p_1, ..., p_n\}$$

- Wanted: translation t and rotation R that minimizes the sum of the squared error:

$$E(R,t) = \frac{1}{N_p} \sum_{i=1}^{N_p} ||x_i - Rp_i - t||^2$$

Where x_i and p_i are corresponding points.

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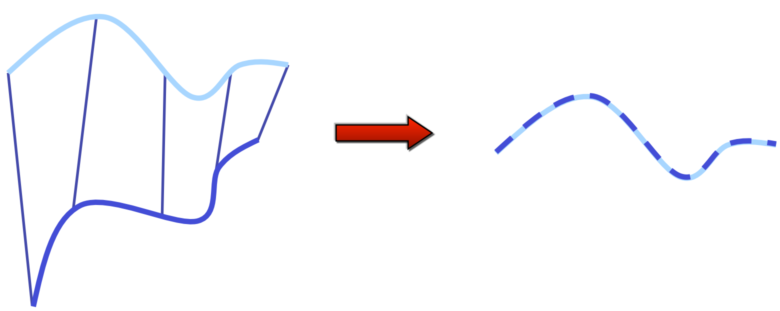
Key Idea

- If the correct correspondences are known, the correct relative rotation/translation can be calculated in closed form.



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Center of Mass

$$\mu_x = \frac{1}{N_x}\sum_{i=1}^{N_x} x_i \quad \text{and} \quad \mu_p = \frac{1}{N_p}\sum_{i=1}^{N_p} p_i$$

are the centers of mass of the two point sets.

Idea:

- Subtract the corresponding center of mass from every point in the two point sets before calculating the transformation.
- The resulting point sets are:

$$X' = \{x_i - \mu_x\} = \{x'_i\} \quad \text{and}$$
$$P' = \{p_i - \mu_p\} = \{p'_i\}$$

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SVD

Let $W = \sum_{i=1}^{N_p} x'_i p'^T_i$

denote the singular value decomposition (SVD) of W by:

$$W = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

where $U, V \in \mathbb{R}^{3\times 3}$ are unitary, and

$\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the singular values of W.

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SVD

Theorem (without proof):

If rank(W) = 3, the optimal solution of E(R,t) is unique and is given by:

$$R = UV^T$$
$$t = \mu_x - R\mu_p$$

The minimal value of error function at (R,t) is:

$$E(R,t) = \sum_{i=1}^{N_p} (||x'_i||^2 + ||y'_i||^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)$$

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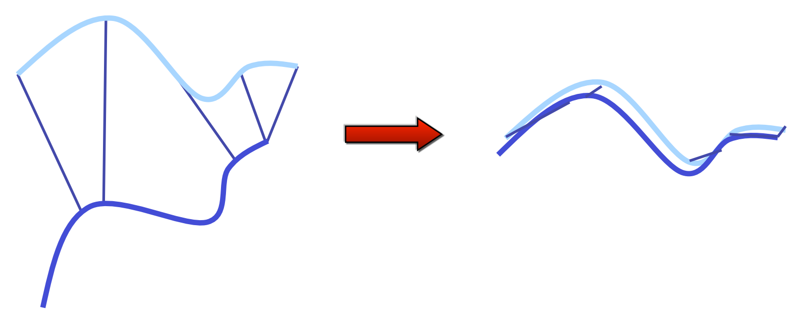
Unknown Data Association

- If correct correspondences are not known, it is generally impossible to determine the optimal relative rotation/ translation in one step



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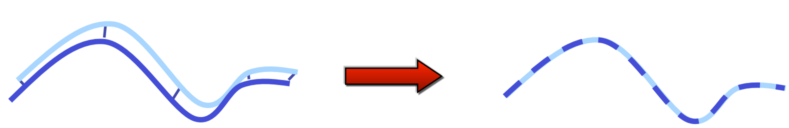
ICP-Algorithm

- Idea: iterate to find alignment
- Iterated Closest Points (ICP) [Besl & McKay 92]
- Converges if starting positions are "close enough"



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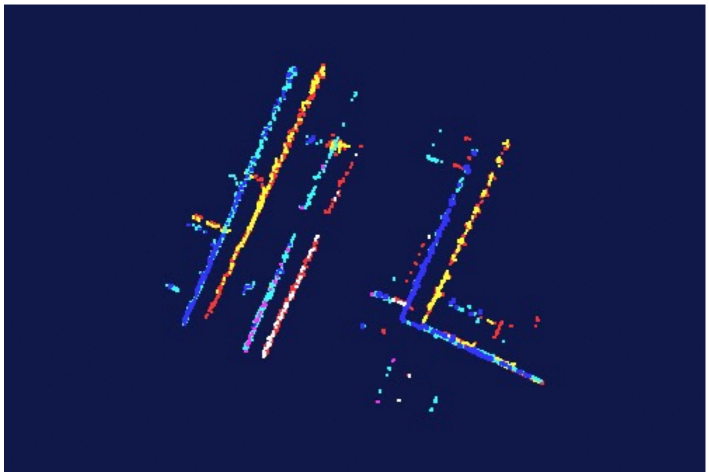
Iteration-Example



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ICP-Variants

- Variants on the following stages of ICP have been proposed:

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

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Performance of Variants

- Various aspects of performance:
  - Speed
  - Stability (local minima)
  - Tolerance wrt. noise and/or outliers
  - Basin of convergence
    (maximum initial misalignment)
- Here: properties of these variants

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ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

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Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based Sampling
- Normal-space sampling
  - Ensure that samples have normals distributed as uniformly as possible

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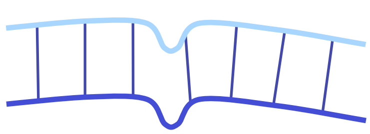
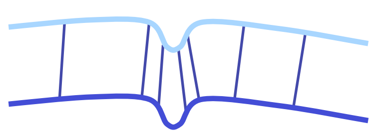
Normal-Space Sampling



uniform sampling



normal-space sampling

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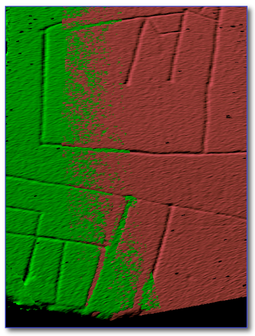
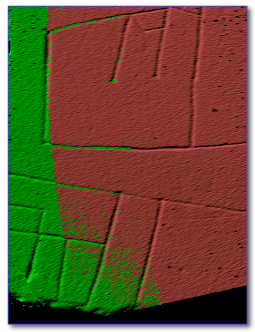
Comparison

- Normal-space sampling better for mostly-smooth areas with sparse features [Rusinkiewicz et al.]



Random sampling



Normal-space sampling

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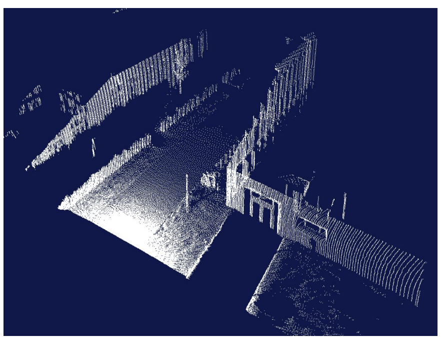
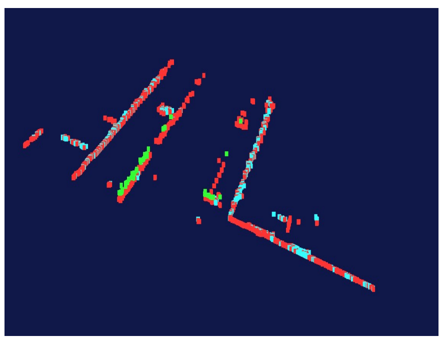
Feature-Based Sampling

- try to find “important” points
- decrease the number of correspondences
- higher efficiency and higher accuracy
- requires preprocessing



3D Scan (~200.000 Points)



Extracted Features (~5.000 Points)

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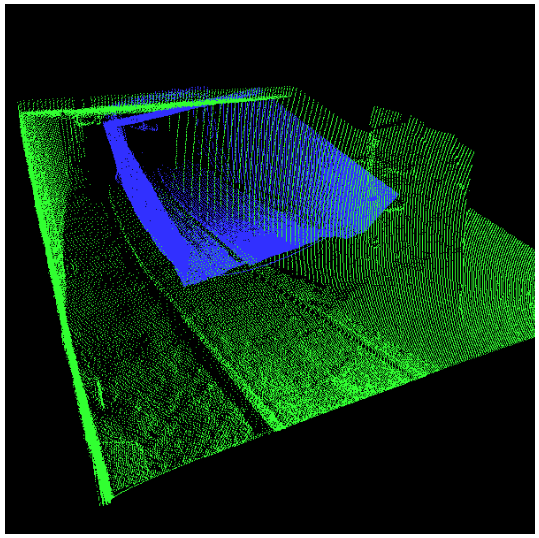
Application



[Nuechter et al., 04]

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ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Data association
4. Rejecting certain (outlier) point pairs

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Selection vs. Weighting

- Could achieve same effect with weighting
- Hard to guarantee that enough samples of important features except at high sampling rates
- Weighting strategies turned out to be dependent on the data.
- Preprocessing / run-time cost tradeoff (how to find the correct weights?)

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ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
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Data Association

- has greatest effect on convergence and speed
- Closest point
- Normal shooting
- Closest compatible point
- Projection
- Using kd-trees or oc-trees

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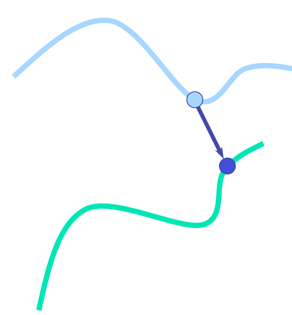
Closest-Point Matching

- Find closest point in other the point set



Closest-point matching generally stable, but slow and requires preprocessing

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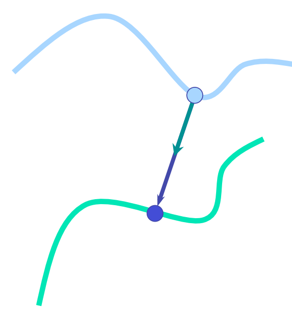
Normal Shooting

- Project along normal, intersect other point set



Slightly better than closest point for smooth structures, worse for noisy or complex structures

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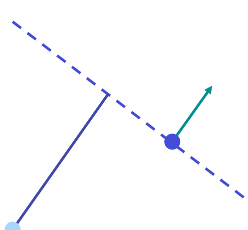
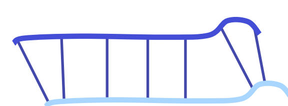
Point-to-Plane Error Metric

- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]





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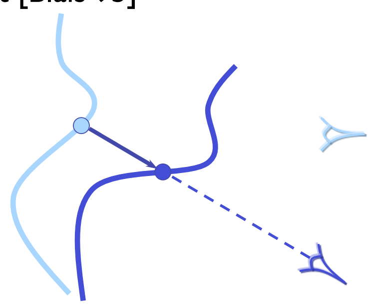
Projection

- Finding the closest point is the most expensive stage of the ICP algorithm
- Idea: simplified nearest neighbor search
- For range images, one can project the points according to the view-point [Blais 95]



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Projection-Based Matching

- Slightly worse alignments per iteration
- Each iteration is one to two orders of magnitude faster than closest-point
- Requires point-to-plane error metric

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Closest Compatible Point

- Improves the previous two variants by considering the compatibility of the points
- Compatibility can be based on normals, colors, etc.
- In the limit, degenerates to feature matching

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ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Nearest neighbor search
4. Rejecting certain (outlier) point pairs

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Rejecting (outlier) point pairs

- sorting all correspondences with respect to there error and deleting the worst t%, Trimmed ICP (TrICP) [Chetverikov et al. 2002]
- t is to Estimate with respect to the Overlap

→ Problem: Knowledge about the overlap is necessary or has to be estimated

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ICP-Summary

- ICP is a powerful algorithm for calculating the displacement between scans.
- The major problem is to determine the correct data associations.
- Given the correct data associations, the transformation can be computed efficiently using SVD.

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